Indian Statistical Institute, Bangalore Centre
J.R.F. (I Year) : 2016-2017
Semester I : Mid-Term Examination
Analysis - I

07.09.2016 Time: $2\frac{1}{2}$ hours. Maximum Marks : 40

Note: Notation and terminology are understood to be as used in class. State clearly the results you are using in your answers.

1. ($8+2=10$ marks) $(\Omega, \mathcal{B}, \mu)$ is a σ-finite measure space. For $E, F \in \mathcal{B}$ define $d(E,F) = \mu(E \Delta F)$.

   (i) Show that $(E,F) \mapsto d(E,F)$ satisfies the triangle inequality.

   (ii) Is d a metric on $\mathcal{B}$?

2. (10 marks) Let $\mathcal{SW} = \{(-\infty, x] \times (-\infty, y] : (x,y) \in \mathbb{R}^2\}$ denote the class of all closed 'south-west regions' in $\mathbb{R}^2$. Show that the Borel $\sigma-$algebra $\mathcal{B}(\mathbb{R}^2)$ is generated by the class $\mathcal{SW}$.

3. (10 marks) Let f_1, f_2, f_3, f_4 be real valued Borel measurable functions on a measurable space $(\Omega, \mathcal{B})$. Let $g(\omega) =$ second smallest among the real numbers $\{f_1(\omega), f_2(\omega), f_3(\omega), f_4(\omega)\}$. Show that g is Borel measurable.

4. (10 marks) For $n = 1, 2, \cdots$ let f_n be a real valued measurable function on a $\sigma-$finite measure space $(\Omega, \mathcal{B}, \mu)$ such that

$$\sum_{n=1}^{\infty} \int_{\Omega} |f_n(\omega)| d\mu(\omega) < \infty.$$

   Show that $\sum_{n=1}^{\infty} f_n$ converges $\mu-$a.e. to a $\mu-$integrable function f, and that

$$\int_{\Omega} f(\omega) d\mu(\omega) = \sum_{n=1}^{\infty} \int_{\Omega} f_n(\omega) d\mu(\omega).$$